Proceedings of the XV<sup>th</sup> Bi-Annual Symposium on Measuring Techniques in Transonic and Supersonic Flows in Cascades and Turbomachines 21-22 September, Firenze/Italy

# A New Method for the Calibration of the Directional Sensitivity of X-Hot-Wire Probes

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# ABSTRACT

The paper presents a new method for the calibration of the directional sensitivity of X-hot-wire probes which bases on the numeric approximation of two polynomials to the measured directional sensitivity and therefore basically differs from the conventional approach which merely uses calibration factors to map the effects of the secondary components of the flow vector. This novel calibration method distinctly improves the measuring accuracy of X-hot-wire probes especially when the flow angle of the flow vector under measurement is subject to wide variations, as would be the case with unsteady highly-turbulent flows. It will, e.g., reduce an approximate 2° error that may well occur in angular measurement performed in accordance with the conventional method to about 0.5°, and when measuring the magnitude of the velocity vector, it will typically reduce an approximate 2% error to about 1%. This clearly exemplifies the advantages provided by the novel method when making measurements of unsteady turbulent flows, such as would occur in the wakes of turbomachinery rotor blades. The novel method also facilitates the manipulation of the X-hot-wire probes during their installation and alignment within the flow. This method, however, will be helpful only when the signals from the X-hot-wire probes are subjected to high-frequency digitization and then processed digitally. This approach, however, has since become a general practice.

The high degree of improvement in the measuring accuracy of X-hot-wire probes achieved through the novel method is further illustrated by way of a measurement made of the wake flow of a model flat plate. This measurement was made, with assistance from MTU, by the Genoa University Department of Fluid Machinery, Energetic Systems and Transportation (DIMSET) under a Brite/Euram project.

### NOMENCLATURE

- a coefficients of polynomial in Eq. (21)
- b coefficients of polynomial in Eq. (22)
- c velocity vector
- C<sub>eff</sub> effective cooling velocity, "sensed" by the hotwire
- C<sub>0</sub> constant in the King's law relationship
- E hot-wire anemometer bridge output voltage

	$c_{eff} = 0$
e′	hot-wire anemometer bridge output signal fluctua-
	tion
f	non-dimensional quantity according to Eq. (10)
h	yaw calibration function defined by Eq. (19)

hot-wire anemometer bridge output voltage at

- k yaw or pitch sensitivity calibration factor
- m exponent in the King's law relationship
- t time

 $E_0$ 

- u, v, w components of velocity vector  $\vec{c}$  in x-, y-, zdirections, respectively
- $U_{\infty}$  local free stream velocity
- x, y, z co-ordinates
- $\alpha$  angle of velocity vector  $\vec{c}$  in the x y plane
- $\alpha_{id}$  yaw calibration function defined by Eq. (18)
- $\alpha_{S}$  angle between hot-wire and x axis
- $\vartheta$  angle of velocity vector  $\vec{c}$  in the x z plane

### **Subscripts**

- B bi-normal
- i hot-wire 1 and hot-wire 2, respectively
- T tangential
- $\infty$  local free stream

#### **Superscripts**

time averaged

fluctuating quantity

# 1. INTRODUCTION

The progress made in the field of the hot-wire measuring technique over the last 15 years is attributable to a large extent to the enormous progress achieved in the field of digital data processing in the same period of time. Only after high-speed digital data recording and software-based data processing systems had become available it was possible, for example, to investigate periodic unsteady flow conditions. Furthermore the use of mini-computers and PCs with specialized application programs, such as transfer functions, linearizations etc., allowed the accuracy of flow measurements with hot-wire probes to be markedly improved. The novel method for calibration of the directional sensitivity of X-hot-wire probes presented here was likewise developed with a view to the possibilities offered by the use of digital data processing techniques (see Schröder (1985)). Being based on an entirely new conceptual approach, this method distinctly improves the measuring accuracy and facilitates the measurements proper.

# 2. DIRECTIONAL SENSITIVITY OF THE X-HOT-WIRE PROBE

If a hot-wire probe – as illustrated in **Figure 1** – is to be used for measuring a three-dimensional flow, its directional characteristics, i.e. its sensitivity in the normal, tangential and bi-normal directions, must be determined first. For this purpose, the velocity vector  $\vec{c}$  is decomposed to obtain the three wire-related components  $c_N$ ,  $c_T$  and  $c_B$  for which Joergensen (1971) suggests the following relationship:

$$c_{\text{eff}}^2 = c_N^2 + k_T^2 \cdot c_T^2 + k_B^2 \cdot c_B^2 \tag{1}$$

where the quantity  $c_{eff}$  is the effective cooling velocity "sensed" by the hot-wire, i.e. a fictitious velocity which – acting on the hot-wire in normal flow direction – would yield the same output voltage as the actual velocity vector  $\vec{c}$ .



Figure 1: Decomposition of the velocity vector into its wire-related components

The coefficients  $k_T$  and  $k_B$  are yaw and pitch sensitivity calibration factors that can either be taken from literature, e.g. Joergensen (1971), or individually determined for each probe. The relationship between the fictitious velocity  $C_{eff}$  and the output voltage of the hot-wire anemometer  $C_{eff} = f(E)$  is determined by means of a velocity calibration which is described in detail in the annex.

If, as shown in **Figure 1**, the hot-wire probe is aligned normal to the main flow direction x, the following wirerelated velocity components are obtained:

$$c_{\mathsf{N}} = \mathbf{c} \cdot \cos \alpha \cdot \cos \vartheta \tag{2}$$

$$c_{\mathsf{T}} = \mathbf{c} \cdot \sin \alpha \tag{3}$$

$$c_{\rm B} = c \cdot \cos \alpha \cdot \sin \vartheta \tag{4}$$

Substitution of  $c_N$ ,  $c_T$  and  $c_B$  in Eq. (1) with Eqs. (2) to (4) results in the so-called cosine law which describes the directional sensitivity of the hot-wire probe and has already been suggested in this form by Hinze (1975) as well as Champagne et al. (1967) (however, for the special case  $\vartheta = 0^\circ$  only).

The fact that in this equation the measured value  $c_{eff}$  is characterized by the three unknown values c,  $\alpha$  and  $\vartheta$  shows that probes with two wire sensors arranged at different angles to the main flow direction, the so-called X-hot-wire probes, have to be used for the measurement of two-dimensional flows and probes with three wire sensors, the so-called triple hot-wire probes for the measurement of three-dimensional flows.

**Figure 2** shows such an X-hot-wire probe for which the novel method for calibration of the directional sensitivity presented here has been developed.



Figure 2: Alignment of the X-hot-wire probe within the flow

The two platinum-plated tungsten wires with a length of 1.75 mm and a diameter of 5  $\mu$ m are aligned, as illustrated in **Figure 2**, symmetrically to the main flow direction x with an angle of  $2\alpha_S$  between them.

By analogy with the Eqs. (2) to (4) the following wirerelated velocity components are obtained for the X-hot-wire probe:

 $c_{Ni} = c \cdot (\cos \alpha \cdot \cos \vartheta \cdot \sin \alpha_{S} \pm \sin \alpha \cdot \cos \alpha_{S}) (5)$ 

 $c_{Ti} = c \cdot (\sin \alpha \cdot \sin \alpha_S \mp \cos \alpha \cdot \cos \vartheta \cdot \cos \alpha_S) (6)$ 

$$\mathbf{c}_{\mathsf{B}i} = \mathbf{c} \cdot \cos \alpha \cdot \sin \vartheta \tag{7}$$

where i = 1, 2 for wire 1 and wire 2, respectively.

Since a two-dimensional flow has to be assumed  $\vartheta = 0^{\circ}$  is defined. Thus, and by inserting Eqs. (5) and (6) in Eq. (1) the known response equations of the X-hot-wire probe are obtained (with  $k_i = k_{Ti}$ ):

$$c_{effi}^{2} = c^{2} \cdot \left[ sin^{2} (\alpha_{S} \pm \alpha) + k_{i}^{2} \cdot cos^{2} (\alpha_{S} \pm \alpha) \right] (8)$$

or, in cartesian coordinates

$$c_{effi}^{2} = (\mathbf{u} \cdot \sin\alpha_{S} \pm \mathbf{v} \cdot \cos\alpha_{S})^{2} + k_{i}^{2} \cdot (\mathbf{v} \cdot \sin\alpha_{S} \mp \mathbf{u} \cdot \cos\alpha_{S})^{2}$$
(9)

Measurements with X-hot-wire probes are generally evaluated by calculating – for a pair of measured values  $C_{effi}$ 

- the components u and v of the two-dimensional velocity vector  $\vec{c}$  using Eqs. (9). This is rather easy if, as in the case of the sampling technique, the pair of measured values Ceffi represents instantaneous values. If, however, there are only time-averaged voltage values  $\overline{\mathsf{E}}_i$  and their RMS values  $\sqrt{e_i'^2}$  or time-averaged velocities  $\overline{c_{effi}}$  and the RMS of the velocity fluctuations  $\sqrt{c'_{effi}^2}$  available, Eqs. (9) would have to be developed either as a power series (see Hinze (1975), Champagne et al. (1967)) which imposes certain limitations as to the maximum possible turbulence intensity, or other methods would have to be used (see Acrivlellis (1977, 1978)) which involve a more complicated and time-consuming measuring effort and can be applied to steady-state flows only. These disadvantages are avoided by the sampling technique where the instantaneous values u and v are calculated first, followed by calculation of the averaged and the RMS values.

Calculation of the instantaneous values u and v from a given pair of values  $C_{effi}$  may, for example, be performed using the following solution of Eqs. (8):

By introducing the non-dimensional quantity f

$$f = \frac{c_{eff1}}{c_{eff2}}$$
(10)

the flow angle  $\alpha$  is calculated from

$$\alpha = \frac{1}{2} \cdot \left[ \arcsin(A_1) + \arctan(A_2) \right]$$
(11)

using the arguments

$$A_{1} = \frac{(1 + k_{2}^{2}) \cdot f^{2} - (1 + k_{1}^{2})}{\sqrt{(C_{1} - C_{2} \cdot f^{2})^{2} \cdot \cos^{2} 2\alpha_{s} + (C_{1} + C_{2} \cdot f^{2})^{2} \cdot \sin^{2} 2\alpha_{s}}}$$

$$A_{2} = \frac{(C_{1} - C_{2} \cdot f^{2}) \cdot \cos 2\alpha_{S}}{(C_{1} + C_{2} \cdot f^{2}) \cdot \sin 2\alpha_{S}}$$

and the constants

$$C_1 = k_1^2 - 1$$
  
 $C_2 = k_2^2 - 1$ 

The magnitude c of the velocity is obtained by substituting  $\alpha$  in one of the Eqs. (8) as follows:

$$c = \sqrt{\frac{c_{\text{eff1}}^2}{\sin^2(\alpha_{\text{S}} + \alpha) + k_1^2 \cdot \cos^2(\alpha_{\text{S}} + \alpha)}}$$
(12)

The components u and v can then be calculated as

$$\mathbf{u} = \mathbf{c} \cdot \cos \alpha \tag{13}$$

$$= \mathbf{c} \cdot \mathbf{sin} \alpha$$
 (14)

As can be seen from **Figure 2**, measurements with an X-hot-wire probe presuppose a flow angle  $\alpha$  which is in the range of  $-\alpha_{\rm S} \leq \alpha \leq \alpha_{\rm S}$ , since otherwise the flow would be directed towards the prongs of the hot-wires and erroneous results would be obtained. Since, furthermore, the Eqs. (8) can be unambiguously solved only in the range  $(-90^{\circ} + \alpha_{\rm S}) \leq \alpha \leq (90^{\circ} - \alpha_{\rm S})$  depending on the probe specific angle  $\alpha_{\rm S}$  the more limiting of the two conditions must be observed.

ν

The accuracy of this solution (Eqs. (11) and (12)) strongly depends on how precisely the true directional sensitivity of the X-hot-wire probe is described by Eqs. (8). To verify this as well as to determine the calibration factors  $k_i$  for the tangential flow around the hot-wires and to align the X-hot-wire probe within the main flow direction, as shown in **Figure 2**, the directional sensitivity of the X-hot-wire probe must be measured. For this purpose, the probe is yawed in increments in a wind tunnel with an undisturbed flow at a constant velocity of  $c = U_{\infty}$ . The velocities  $c_{effi}$  acting on the two wire sensors are then determined and plotted – normalized with the maximum value at zero yaw  $c_{eff max} = c$  – versus the yaw angle  $\alpha$ ; see **Figure 3**. These distributions are compared with the response equations, Eqs. (8), which are transformed as follows:

$$\frac{c_{effi}}{c} = \sqrt{\sin^2(\alpha_S \pm \alpha) + k_i^2 \cdot \cos^2(\alpha_S \pm \alpha)}$$
(15)  
i = 1, 2



Figure 3: Measured directional sensitivity of an X-hot-wire probe Dantec 55P63 compared with the cosine law (Eqs. (15) and the polynomial approximation (Eqs. (24) and (25))

The calibration factors  $k_i$  are calculated by linking Eqs. (15) to the effective velocities  $c_{effi}$  measured at the points

 $\alpha = \mp \alpha_S$ . The angle  $2\alpha_S$  is measured under the microscope since – for manufacturing reasons – the two wires of the X-hot-wire probe are not exactly perpendicular to each other.

**Figure 3** shows the measured directional sensitivity of an X-hot-wire probe Dantec 55P63 in comparison with Eqs. (15). The constants used in this case were

$$\alpha_{\rm S} = 43.08^{\circ}$$
  
 $k_1 = 0.1591$   
 $k_2 = 0.1389$ 

The deviations of the measured values from the distributions determined using Eqs. (15) that can be seen in **Figure 3** result from the fact that these equations do not exactly reflect the directional sensitivity of the X-hot-wire probe in skewed flow. It is also not possible to eliminate these deviations by calculating the calibration factors  $k_i$  for other angular positions.

The errors in the calculation of the unknown values  $\alpha$  and c resulting from the above deviations are illustrated in **Figures 4** and **5**. The error  $\Delta \alpha$  is calculated as the difference between the yaw angle of the probe support  $\alpha_{PS}$ , which corresponds to a virtual flow angle at the time the directional sensitivity is measured, and the flow angle  $\alpha$  determined from the measured values using Eq. (11):

$$\Delta \alpha = \alpha_{\mathsf{PS}} - \alpha \tag{16}$$

The error  $\Delta c$  is calculated from the difference between the magnitude c of the flow velocity determined using Eq. (12) and the local free stream velocity  $U_{\infty}$  calculated from the pneumatic stagnation pressure measurement:

$$\Delta c = \frac{c - U_{\infty}}{U_{\infty}} \cdot 100 \tag{17}$$



Figure 4: Error in calculating the flow angle α
a) using the cosine law (Eq. (11))
b) using the polynomial approximation (Eq. (21))

While the error  $\Delta c$  reaches a value of 2.4% max. the error  $\Delta \alpha$  is approx. 10% in the range of  $\alpha = \pm 20^{\circ}$ . This relatively high value can be reduced by describing the distribution shown in **Figure 4**,  $\Delta \alpha = f(\alpha)$ , by an empirical polynomial or sine formulation which then can be used to correct

the flow angle  $\alpha$  determined using Eq. (11). The distribution  $\Delta c = f(\alpha)$ , however, which is shown in **Figure 5**, does not lend itself to approximation using such a correction function. In view of the rather low maximum possible error value, correction is not absolutely necessary here, but the possibilities of reducing the remaining error by approximation of the curve  $\Delta \alpha = f(\alpha)$  are limited as a result.





For this reason a new evaluation method was developed which entirely departs from the basic Eq. (1) and thus also from the cosine law (Eqs. (15)). This method designated as polynomial approximation, which is described in the following, markedly improves conformity with the measured directional sensitivity – as can be seen from **Figures 3**, **4** and **5** – and thus results in markedly smaller errors  $\Delta \alpha$  and  $\Delta c$ .

# 3. APPROXIMATION OF THE DIRECTIONAL SEN-SITIVITY OF THE X-HOT-WIRE PROBE

Eq. (1) which is frequently used to describe the directional sensitivity of a hot-wire probe is based on geometrical considerations, i.e. the velocity vector  $\vec{c}$  acting on the wire is decomposed into three wire-related components and the components acting in the tangential and bi-normal directions are weighted using calibration factors. The fact that the measured directional sensitivity is not adequately reflected by this equation might be attributable to disturbance effects, such as effects of the probe on the flow. These disturbance effects can at the most be compensated by empirical correction functions which have to be approximated to the error distribution determined for the flow angle  $\Delta \alpha = f(\alpha)$ , see **Figure 4**. Therefore, it suggests itself to entirely depart from this description of the directional sensitivity and follow a numeric approximation approach instead.

The newly developed evaluation method, therefore, bases on the numeric approximation of two polynomials to the measured directional sensitivity. It allows the two values to be calculated, C and  $\alpha$ , to be represented independently of each other as functions of the given values C<sub>effi</sub> which is not possible with the Eqs. (15). The ideal case would be polynomials that allow the values C and  $\alpha$  to be calculated explicitly as functions of the input values C<sub>effi</sub> or as functions of the non-dimensional quantity f defined in Eq. (10).

The distribution of  $\alpha$  versus f shown in Figure 6, however, is rather unsuitable for approximation as f assumes values between 0.2 and 1 in the range  $\alpha = -40^{\circ}$  to  $\alpha = 0^{\circ}$  and values between 1 and 6 in the range  $\alpha = 0^{\circ}$  to  $\alpha = 40^{\circ}$ . Due to the steep gradient of the distribution in the range of  $f \le 1$  the use of a power series as polynomial for approximation is not recommendable, as this would result in large errors in the calculated flow angle  $\alpha$ , if even only small measuring errors occur.

For this reason a function  $\alpha = f(\alpha_{id})$  is selected for approximation, i.e. the flow angle  $\alpha$  is plotted versus a so-called "ideal" angle  $\alpha_{id}$  which is calculated as follows:

$$\alpha_{id} = \arctan \frac{c_{eff1}}{c_{eff2}} - 45^{\circ}$$
(18)



Figure 6: Measured directional sensitivity of an X-hot-wire probe Dantec 55P63 represented as  $\alpha = f(c_{eff1} / c_{eff2})$ 

The designation "ideal" angle results from the fact that the measured flow angle becomes  $\alpha = \alpha_{id}$  if the directional sensitivity of the X-hot-wire probe is "ideal", i.e. with calibration factors  $k_1 = k_2 = 0$  and a wire angle  $\alpha_S = 45^\circ$ . Under these conditions Eq. (18) results from the Eqs. (8). As can be seen from **Figure 7** the distribution  $\alpha = f(\alpha_{id})$  only slightly deviates from a 45° straight line and thus lends itself to approximation. The deviations from this straight line reflect the difference between actual and ideal directional sensitivity of the X-hot-wire probe. If the function  $\alpha = f(\alpha_{id})$  is  $\neq 0$  for  $\alpha_{id} = 0$ , as shown in **Figure 7**, the X-hot-wire probe is not aligned exactly symmetrical with the main flow direction. But with this evaluation method this is no longer necessary, and the requirement for measuring the wire angle  $\alpha_S$ under the microscope can likewise be waived.

With regard to the second approximation polynomial the introduction of a non-dimensional quantity h instead of the function c = f(f) proved to be adequate, the distribution of which is also approximated as a function of  $\alpha_{id}$ , i.e.  $h = f(\alpha_{id})$ . This quantity is defined by

$$h = \frac{c^2}{c_{\text{eff}1}^2 + c_{\text{eff}2}^2}$$
(19)

It may be interpreted as the ratio of ideal and actual directional sensitivity of the X-hot-wire probe. The following equation results when Eq. (8) is inserted in Eq. (19):

$$h = \frac{1}{\sin^2(\alpha_{\rm S} + \alpha) + \sin^2(\alpha_{\rm S} - \alpha) + k_1^2 \cdot \cos^2(\alpha_{\rm S} + \alpha) + k_2^2 \cdot \cos^2(\alpha_{\rm S} - \alpha)}$$
(20)

For  $k_1 = k_2 = 0$  and  $\alpha_S = 45^\circ$  h becomes 1. As can be seen from **Figure 7** the function  $h = f(\alpha_{id})$  produces a curve which deviates only slightly from a horizontal straight line and is well suited for approximation. The deviations from the straight line once again reflect the difference between actual and ideal directional sensitivity of the X-hot-wire probe.



Figure 7: Measured directional sensitivity of an X-hot-wire probe Dantec 55P63 compared with the results of the polynomial approximation (Eqs. (21) and (22))

The distributions of **Figure 7**,  $\alpha = f(\alpha_{id})$  and  $h = f(\alpha_{id})$ , were approximated as fifth order polynomials to the measured directional sensitivity using the least squares fit method. In practice, higher order polynomials did not prove useful. Calculation of the components u and v of a two-dimensional velocity vector  $\vec{c}$  from two "sensed" velocities  $C_{eff1}$  and  $C_{eff2}$  obtained from measurements is then performed using the approximated functions:

$$\alpha = \sum_{j=0}^{n} a_j \cdot \alpha_{id}{}^j \tag{21}$$

$$h = \sum_{i=0}^{n} b_j \cdot \alpha_{id}{}^j$$
 (22)

where 
$$\alpha_{id} = \arctan \frac{c_{eff1}}{c_{eff2}} - 45^{\circ}$$
 (18)

The magnitude of the flow velocity c is calculated using Eq. (19):

$$c = \sqrt{h \cdot \left(c_{eff1}^2 + c_{eff2}^2\right)}$$
(23)

The components U and V result from

$$u = c \cdot \cos \alpha \tag{13}$$

$$v = c \cdot \sin \alpha$$
 (14)

The coefficients of the polynomials (21) and (22) used for the exemplary distributions in **Figure 7** are listed in **Table 1**.

Coefficients of the polynomials			
	$\alpha(\alpha_{id})$	$h(\alpha_{id})$	
j	aj	bj	
0	-0.7767	1.0168	
1	0.9148	-0.9873 x 10 <sup>-3</sup>	
2	0.2087 x 10 <sup>-2</sup>	-0.2848 x 10 <sup>-4</sup>	
3	-0.5747 x 10 <sup>-4</sup>	-0.4187 x 10 <sup>-6</sup>	
4	-0.3065 x 10 <sup>-5</sup>	-0.2463 x 10 <sup>-7</sup>	
5	0.1928 x 10 <sup>-6</sup>	0.1448 x 10 <sup>-8</sup>	

# Table 1: Coefficients of the exemplary polynomial distributions in **Figure 7** (Eqs. (21) and (22))

As can be seen from **Figures 4** and **5** this evaluation method distinctly improves the measuring accuracy for twodimensional flows. The approximations are performed for the range  $-40^{\circ} \le \alpha \le 40^{\circ}$ , since a larger range would adversely affect the quality of the approximations and the range selected is considered to be by far sufficient for the measurements.

Using Eqs. (21) and (22) to calculate c and  $\alpha$  will result in somewhat more comprehensive computations in cases where distributions of the form  $c_{effi}/c = f(\alpha)$  are to be calculated. Since the representation of the directional sensitivity of X-hot-wire probes as per **Figure 7** supersedes the previous one shown in **Figure 3**, these computations are required only if – as in **Figure 3** – a comparison with distributions calculated using Eqs. (15) is to be made. Eqs. (18) and (23) result in:

$$\frac{c_{eff1}}{c} = \frac{tan(\alpha_{id} + 45^{\circ})}{\sqrt{h \cdot (tan^2(\alpha_{id} + 45^{\circ}) + 1)}}$$
(24)

$$\frac{c_{eff2}}{c} = \frac{1}{\sqrt{h \cdot (tan^2(\alpha_{id} + 45^\circ) + 1)}}$$
(25)

For a given flow angle  $\alpha$ ,  $\alpha_{id}$  is calculated from Eq. (21) using, for example, the Newton iteration method, and the non-dimensional quantity h is determined by means of Eq. (22). These two values are inserted in Eqs. (24) and (25). As can be seen from **Figure 3** the distributions thus calculated coincide much better with the measured distributions than those calculated by means of Eqs. (15).

#### 4. APPLICATION OF THE NOVEL METHOD

The example of an application of the novel method for calibration of the directional sensitivity of X-hot-wire probes which is described below has been part of flow investigations performed by the Genoa University Department of Fluid Machinery, Energetic Systems and Transportation (DIM-SET), with technical assistance from MTU, within the scope of the Industrial and Material Technology Aeronautics Research Project AER2-CT92-0048 "Experimental and Numerical Investigation of Time Varying Wakes behind Turbine Blades" (see Sieverding et al. (1999)).

**Figure 8** is a schematic representation of the measurement performed. A Dantec 55P64 X-hot-wire probe is mounted downstream of a flat plate with rounded trailing edge (thickness D = 16 mm, chord length l = 300 mm) vertically positioned at the height of the flat plate centerline (y/D = 0) and at a distance of x/D = 1.875 downstream. This measurement was part of a comprehensive experimental investigation into the formation of von Kármán vortices in the wakes of model test bodies and airfoils (Sieverding et al. (1999)).



Figure 8: Scheme of the measurement arrangement, X-hotwire probe at y/D = 0, x/D = 1.875

One of the results of the signal evaluation, i.e. time traces of the instantaneous flow angle  $\alpha$ , is illustrated in **Figure 9**. The fluctuations of the flow angle  $\alpha$  attributable to the von Kármán vortices are clearly visible. Furthermore, the distribution  $\alpha(t)$  contains short-time peaks reaching maximum values of up to 60° on the positive side and a bit less distinct peaks of more than 45° on the negative side. As the sampling technique always uses a low-pass filter this phe-

nomenon cannot be attributable to high-frequency disturbances. Nevertheless, there must be an error since the acceptable measuring range of an X-hot-wire probe is in the region of  $\pm 45^{\circ}$ , depending on the size of  $\alpha_{S}$  (see the explanations in section 2 of this paper).





Further investigations then showed that the phenomenon was due to a numeric problem in the evaluation method used. Instead of the conventional method for calibration of the directional sensitivity of X-hot-wire probes as described in section 2, DIMSET had used a self-developed improved variant of this method to enhance the measuring accuracy. In this improved method, the empirical calibration factors  $k_{Ti}$ were determined at all angular positions where calibration measurements were made and not only at the positions  $\alpha = \pm \alpha_{\rm S}$  (see Figure 3). Thus two distributions  $k_{\rm Ti} = f(\alpha)$ are obtained instead of the two factors  $k_{Ti}$ . For these two distributions numeric approximations were made, and the evaluation method then had to determine the unknown values c and  $\alpha$  iteratively, starting with a guess for  $\alpha$ , because from a measurement the two values of Ceffi are obtained only and the value of  $\alpha$  for determining the calibration factors  $k_{Ti}$  is not known yet at this point in time. Although this method has already proven its suitability, there are obviously problems in finding realistic solutions in the extremities of the X-hot-wire probe's measuring range. As recommended by MTU, therefore, DIMSET applied the new evaluation method for the directional sensitivity of X-hot-wire probes described in section 3. The results of the signal evaluation using this new method are illustrated in Figure 10.



Figure 10: Time traces of α(t) in the wake of a rounded trailing edge flat plate, X-hot-wire probe signals evaluated with the novel method

As can be seen, the spike-type deflections described above have disappeared, and the signal fluctuation of the flow phenomenon investigated remains clearly within the boundaries of the probe's measuring range. Within this range the signal traces of **Figures 9** and **10** are in good agreement which suggests that the variant developed by DIMSET already yields a higher accuracy than the pure conventional method. Owing to the problems in the extremities described above and the fact that the iterative method of calculation may be somewhat tedious, in particular in the case of comprehensive measurements in unsteady flows, this method, however, does not have any advantages over that described in section 3.

#### 5. CONCLUSIONS

In comparison with the conventional variant the new method for the calibration of the directional sensitivity of Xhot-wire probes described in this paper distinctly improves the achievable measuring accuracy and also facilitates the manipulation of the X-hot-wire probes during their installation and alignment within the flow. There are no disadvantages as compared with the conventional method since digital data acquisition and processing (sampling technique), a prerequisite for the application of this method, have long become a general practice. The advantages of the new method become particularly evident when unsteady, highly turbulent flows are to be investigated. Therefore, the further development of this method for the calibration of the directional sensitivity of triple hot-wire probes for the measurement of unsteady threedimensional flows will be the next logical step.

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# ANNEX: VELOCITY CALIBRATION OF THE X-HOT-WIRE PROBE

The relationship between the velocity  $c_{eff}$  "sensed" by the hot-wire sensor and the output voltage E of a hot-wire anemometer,  $c_{eff} = f(E)$ , which is needed for the evaluation of hot-wire measurements, is established using the so-called King's law (see Wuest (1969) which describes – in good approximation – the two-dimensional heat transfer from a cylinder in an incompressible flow. From this law, the following relationship between the flow velocity and the electrical power fed to the hot-wire probe can be derived:

$$\mathbf{c}_{\text{eff}} = \left( \mathbf{E}^2 - \mathbf{E}_0^2 \right)^m \cdot \mathbf{C}_0$$

The constants  $E_0$  and  $C_0$  as well as the exponent m are determined by calibration. For the purpose, the probe is exposed to a flow with varying velocities, and the voltage values E supplied by the hot-wire anemometer are recorded for velocity values  $C_{effi}$  determined from pneumatic measurements of the dynamic head. Figure A.1 shows an exemplary calibration curve for an X-hot-wire probe. The different distributions result from the fact that the electrical resistance of the two wire sensors is not identical for manufacturing reasons.



Figure A.1: Exemplary calibration curves for an X-hot-wire probe (Dantec 55P63)

The calibration data are recorded using a data-processing system. For data recording the wind tunnel is run up continuously from zero to a flow velocity that is slightly higher than that selected for the measurements. Whenever the dynamic head increases by a given amount the data-processing system records the voltages of the two hot-wire sensors of the probe and at the same time the dynamic head, the static pressure and the air temperature. The three latter quantities are used to determine the flow velocity  $U_{\infty}$ , and the velocities  $C_{effi}$  "sensed" by the hot-wire probe are then calculated from Eqs. (24) and (25), with  $c = U_{\infty}$  and  $\alpha = 0^{\circ}$ . During the few minutes it takes the wind tunnel to reach the desired velocities more than 150 calibration data points can thus be recorded for each of the two hot-wire sensors. For the sake of

clarity, therefore, **Figure A.1** does not show all measuring points actually recorded in the range above  $c_{eff} = 3 \text{ m/s}$ .

The constants  $E_{0i}$  and  $C_{0i}$  as well as the exponents  $m_i$  for the calibration curves calculated using Eq. (A.1), which are likewise shown in **Figure A.1**, are determined from the measured data points by means of regression calculations. Thus, the functions  $c_{effi} = f(E_i)$  are obtained which are needed for evaluating measurements made using X-hot-wire probes and which can well be approximated because of the large number of measured data points. Since instantaneous voltage values only are recorded by the sampling technique and the (ensemble or time) average and the RMS values are determined by way of calculation, the use of linearizers to establish a linear relationship between flow velocity and output voltage is not required.