Paper 11

HIGH FREQUENCY TEMPERATURE MEASUREMENTS IN ROTATION

R. Dénos, C.H. Sieverding
Von Kármán Institute for Fluid Dynamics
Belgium

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R. Dénos and C.H. Sieverding
von Karman Institute For Fluid Dynamics
1640 Rhode Saint Genèse, Belgium

Abstract

The paper first describes the fundamentals of cold wire resistance thermometry. A new method for the description of complicated transfer functions describing both the prongs and wire frequency response is proposed. The experimental part of the paper starts with an investigation of the transfer function of various probes differing by the wire diameter, the l/d ratio and the wire-prong connection using two simple methods: (1) electrical heating of the wire by a sine current and (2) a temperature step test consisting in injecting the probe into a hot air stream. The first test provides information on the wire response whereas the second serves to study wire prong heat conduction effect. The tests cover a wide range of velocities and densities. A frequency bandwidth of 2 kHz is obtained with a 2.5 μm wire probe at an air velocity of 200 m/s at atmospheric pressure. A numerical compensation system allows to extend the use of this probe to much higher frequencies. Finally, the probe is mounted onto a wheel in a high speed rotating test rig allowing probe traverses through a stationnary hot air jet. It is demonstrated that using the numerical compensation method, it is possible to reconstruct the hot jet temperature profile at frequencies up to 6 kHz.

Nomenclature

I	intensity [A]
Q	heat flux [W]
R	resistance [Ω]
T	temperature [K]
c_p	heat storage capacity [J/(kg.K)]
f	frequency [Hz]
h	heat exchange rate by convection $[Wm^{-2}K^{-1}]$
k	conductivity [W/(m.K)]
t	time [s]
u	peripheral speed [m/s]
v	velocity [m/s]
w	relative speed [m/s]
d_w	wire diameter [m]
l_w	wire length [m]
A_w	$\frac{\pi d_w^2}{4}$ $[m^2]$ wire cross sectionnal area.
S_w	$\pi d_w l_w$ $[m^2]$ wire area available for convection
V_w	$\frac{\pi d_w^2 l_w}{4}$ wire volume
Re	$\frac{\rho v_a d_w}{\mu_a}$ Reynolds number
Nu	$\frac{h d_w}{k_a}$ Nusselt number
α	temperature coefficient $\frac{1}{R_0}\frac{\partial R}{\partial T}$ $[K^{-1}]$
τ	time constant [s]
σ_0^{-1}	resistivity $[\Omega m]$
ρ	density $[kg/m^3]$
μ	dynamic viscosity [kg/(m s)] [Pl]
ω	pulsation [rad/s] $\omega = 2\pi f$

Subscripts

0:	total
s:	static
a:	air
p:	prong
w:	wire

Introduction

The present paper is a feasability study for the use of a cold wire probe for unsteady temperature measurements in turbomachines. The velocity range is one order of magnitude higher than in all previous low speed studies and, depending on the engine size, blade passing frequencies are typically of the order of 2 to 20 kHz. The upper frequency range is obviously not accessible to cold wire probes. The authors' interest was in the lower frequency range as illustrated by the following example: a transonic model turbine of 790 mm tip diameter with 43 guide vanes and 65 rotor blades running at 6500 RPM in a short duration tunnel with 0.5 secondes running time. A cold wire probe mounted onto the rotor blade would experience blade passing frequencies of the order of 4.6 kHz.

Simulation of the unsteady probe behaviour with a numerical system

The main heat transfer mode is the forced convection due to the air flowing around the wire surface:

$$Q_h = -hS_w(T_w - T_a) \tag{1}$$

The frequency response of the wire is mainly due to its thermal inertia:

$$Q_s = V_w \rho_w c_w \frac{\partial T_w}{\partial t} \tag{2}$$

The wire is fastened onto prongs which have a large thermal inertia. The prongs will not be able to follow high frequency fluctuations and a conduction phenomenon will take place between the wire and the prongs; T_p is a boundary condition of:

$$Q_k = -k_w A_w l_w \frac{\partial^2 T_w}{\partial x^2} \tag{3}$$

In the above equations, it is assumed that the gradient in the radial direction of the wire can be neglected because the wire diameter remains small compared to the length.
The unsteady equation including conduction is derived from $Q_s = Q_h + Q_k$:

$$\frac{\partial T_w}{\partial t} = \alpha \frac{\partial^2 T_w}{\partial x^2} - \frac{1}{\tau}(T_w - T_a) \tag{4}$$

where $\alpha = \frac{k_w}{\rho_w c_w}\ W/m^2$ and $\tau = \frac{d_w \rho_w c_w}{4h}\ s$.
In pratice, the wire temperature is measured by the change of the wire resistance which is linked to the temperature through:

$$R_w = R_0(1 + \alpha_w(T_w - T_0)) \tag{5}$$

The wire resistance is measured in a Wheatstone bridge (see Fig. 1) through the voltage V_{out}. The current across the wire is very small and Joule effect will be neglected.
Eq. 4 can be discretized with a Crank-Nicholson scheme. Instantaneous temperature profiles have been calculated and the transfer function was established assuming the prongs were behaving like a first order system.

This equation was found to have the same behaviour in the time as well as in the frequency domain as the combination of two first order equations discretised as follows:

$$\frac{T_w(n+1) - T_w(n)}{\Delta t} = \frac{1}{\tau_w}(T_w(n) - T_a(n)) \tag{6}$$

$$\frac{T_p(n+1) - T_p(n)}{\Delta t} = \frac{1}{\tau_p}(T_p(n) - T_a(n)) \tag{7}$$

$$T(n+1) = gT_w(n+1) + (1-g)T_p(n+1) \tag{8}$$

where n is such that $t = n * \Delta t$.
The transfer function of a first order system is well known and can be written as:

$$H(\omega) = \frac{1}{1 + j\omega\tau} \tag{9}$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega\tau)^2}} \tag{10}$$

$$\hat{H(\omega)} = atan(-\omega\tau) \tag{11}$$

where $|H(\omega)|$ and $\hat{H(\omega)}$ are the corresponding gain and phase and $w = 2\pi f$ is the pulsation.
Another characteristic value for a first order equation is the cut-off frequency defined as the frequency for which $|H(\omega)| = \frac{\sqrt{2}}{2} = 0.707$. A straight forward relationship exists between the cut-off frequency and the time constant, namely:

$$f_{cut} = \frac{1}{2\pi\tau} \tag{12}$$

Details on first order system and their numerical discretisation can be found in Oppenheim (1983) or Rabiner (1975). Starting from an initial guess of T_w and T_p at n=1, Eq. 6 allows to compute $T_w(n+1)$ and $T_p(n+1)$, the resulting probe temperature being $T(n+1)$. The transfer function can be derived analytically from Eq. 9 and is shown in Fig. 2. The frequency response of the wire without conduction effect is plotted as a dotted line; the combination of the two first order equations is plotted as a continuous line. At low frequency, both prongs and wire can follow the temperature fluctuations; then the prongs begin to lag behind until a plateau is reached where only the wire responds. The plateau results from a large difference between the thermal capacity of the prongs and that of the wire. The plateau level depends on the relative importance of conduction with respect to convection in the thermal balance and is denoted by g in Eq. 6. Increasing l/d or the velocity would raise this level. Finally, at high frequencies, the wire also lags behind.
Extending this numerical analysis to N first order systems allows a great freedom in the description of the transfer function shape, unlike the simple analytical expression of Paranthoen et al. (1982) or the complex expression obtained by Tsuji et al. (1992) which is limited to three first orders. Moreover, the linear system formed by those N equations can easily be reversed to compute T_a from a measurement of T_w and realise in this way a simple compensation system. Unlike the electronic compensation (RC filter) described by Weeks et al. (1988), the numerical system allows to describe complicated transfer functions with an easy tuning through an adequate choice of the various time constants. The global transfer function is obtained by summing in the complex domain the transfer functions of the N first orders (see Eq. 9) weighted by their respective contributions (The Laplace transform is linear).

Experimental determination of the probe transfer function

Wire response

The wire frequency response was evaluated by the electrical heating method described by LaRue et al. (1974). A frequency generator is used to inject a sine current into the bridge. This current heats the wire and the bridge output voltage relates its temperature change. The gain is computed by dividing the output voltage obtained for a given frequency by a reference voltage, measured at a frequency sufficiently low to guaranty that the wire is able to follow perfectly the heat fluctuations. It was observed that a too large AC current amplitude resulted in an ouput voltage due, not only to the wire temperature change, but also to an electrical response of the bridge, especially if it was not set at equilibrium before. In order

to minimise this electrical response, a small AC current was superimposed onto a DC current for which the bridge was equilibrated. In Fig. 3 the gain in deciBell ($G(dB) = 20log_{10}(G)$) is plotted in function of the frequency of the AC component. Provided the AC component remains small, all data points fall onto the same line. The gain corresponding to a first order transfer function is plotted as a continuous line and confirms the behavior predicted by the theory. Notice that this method only provides information on the wire response; conduction effects exist but do not affect the cut-off frequency of the wire.
The influence of the velocity on the cut-off frequency at atmospheric pressure is presented in Fig. 4 for two wire diameters ($5\mu m$ and $2.5\mu m$). A theoretical cut-off frequency is also plotted and is much higher than the measured values. This was also observed by LaRue et al.(1974), Hojstrup et al.(1976) and Weeks et al.(1988); the discrepancy tends to increase when going to smaller diameters. Hojstrup et al. even mention that in still air, no clear improvement is felt between wire diameters of $1.0\mu m$ and $0.2\mu m$. Notice that the correlation of Collis and Williams (1959) was developped for constant temperature hot-wire purposes whereas the present test conditions are unsteady. However, the experimental data show the same trends as predicted by the theory: higher velocity improves convection and increases the frequency response; a smaller diameter reduces the thermal inertia of the wire and increases the frequency response. A 2kHz cut-off frequency is achieved at 200 m/s with the $2.5\mu m$ wire at atmospheric pressure.

Prong response

The response of the prongs to temperature variations is several orders of magnitude slower than that of the wire. Their behaviour is best investigated in a temperature step test. To this end, the probe was injected from air at ambient temperature into a hot jet exiting from a 12 mm diameter nozzle; the temperature difference was typically 50^oC. The probe used for this test had a $2.5\mu m$ wire mounted onto 0.4 mm diameter stainless steel prongs. The probe response is plotted in Fig. 5. A zoomed view of the step area shows that the wire responds very fast whereas the prongs need 10 s to reach the jet temperature. Two methods were used to compute the transfer function. For both, a numerical reconstruction of the true step is needed. The first method consists in performing a FFT on the reconstructed step and on the original wire response. The ratio of the two FFT gives the complex transfer function from which the gain is computed (see Fig. 5.a). The second method uses a numerical system comprising five first order systems. The time constant and the contribution of each first order system to the overall signal are adapted so that the system response to the reconstructed step fits the experimental response. The match with the FFT result is satisfactory and the transfer function of the wire is added as shown on Fig. 5.b (the cut-off frequency is known from the electrical heating test). Four first order systems were used to simulate the behaviour of the prongs but only one is needed for the wire behaviour.

Tests in rotation, compensation

A rotating model test rig was built to test the cold wire probe under conditions similar to those in the transonic turbine stage referred to in the introduction. The set-up is depicted in Fig. 6. The probe is fixed onto a rotating wheel at a radius of 380 mm. Instead of traversing a thermal wake issuing from a cooled turbine guide vane, the probe is rotated through a heated air jet exiting from a 12 mm diameter nozzle. At a distance of 8 mm from the nozzle exit, the cold wire sees a trapezoidal temperature profile with a thermal shear layer thickness of approximately 1.2 mm on either side of the jet. The temperatures outside and inside the jet are controlled by thermocouples. As for the step tests, the temperature difference between the ambient air and the heated jet was typically 50 K. An opto-electronic data transmission system (Sieverding et al. (1992)) is used to transmit the signal from the rotating frame to a fixed data acquisition system. The tests were run at atmospheric pressure. For all tests, the probe was aligned within $\pm 10^o$ with the relative flow direction resulting from the jet velocity v in the axial direction and the peripheral speed u of the probe. The probe measures a total relative temperature: $T_{0r} = T_s + \frac{w^2}{2c_p}$ where $w = \sqrt{v^2 + u^2}$. This temperature can be compared with the relative temperature derived from the thermocouple measurements through $T_{0r} = T_0 + \frac{u^2}{2c_p}$. Notice that $T_{0r} \neq T_0$ but $\Delta T_{0r} = \Delta T_0$. For a given rotational speed, the jet velocity is adjusted in order to obtain the desired relative velocity.
Typical measurements are shown in Fig. 7 for several rotational speeds but at a constant relative velocity of 200 m/s. Several jet traverses were recorded and a phase locked average was applied (from 1 traverse at

3.2 Hz up to 50 traverses at 6000 Hz). The trapezoidal shape represents a simplified jet shape (dot-dashed lines). The frequency indicated for each experiments corresponds to the frequency of a periodic trapezoidal signal which would fit the jet shape. At low frequency (3.2 Hz), the wire responds immediately and the slight slope on the top level indicates that the prongs are warming up. At 300 Hz, the wire response is still satisfactory but the prongs do not have enough time to heat up significantly (plateau region in the transfer function in Fig 7). At 980 Hz, the wire itself cannot follow anymore the temperature change. At higher frequencies (4600 Hz and 6000 Hz) the signal is completely distorted. The 6000 Hz signal is obtained at 4000 RPM. The rotational speed was raised to 5000 RPM (upper limit of wheel speed) without breaking the wire. The corresponding jet passing frequency is 8 kHz but the signal is severly affected by the noise.
The upper left curve in Fig. 7 represents the transfer function determined by electrical heating and step tests. For each experiment, the gain was computed from the cold wire signal and the temperatures measured by the thermocouple and plotted as dots on the same figure. The agreement is good although the results are obtained with two distinct probes (but which have the same characteristics).
In order to correct the recorded signals for prong-wire interaction and wire roll-off at high frequency, the numerical compensation system described previously is used. The transfer function of the probe and that of the compensator are sketched in the upper left curves in Fig. 8. Injecting the measured wire temperature T_w into the numerical compensation system provides immediately the air temperature T_a. Compensated signals are plotted in Fig. 8 as dashed lines. The compensation system seems to be efficient across the entire frequency domain.

Conclusions

1. The present investigation demonstrates that the cold wire resistance thermometry technique is of potential interest for time resolved temperature measurements in turbomachines with blade passing frequencies of several kHz. Adequate compensation systems allow to treat temperature signals of frequencies three times as high as the cut-off frequency.
2. The accuracy of the measurements depends to a large extend on the precise determination of the prong-wire transfer function over the entire frequency domain. The transfer function can adequately be described by a numerical system based on the combination of N first orders schemes. This scheme lends itself also readily for numerical compensation.
3. A probe with a wire diameter of $2.5\mu m$ and 0.4 mm active length (prong to prong) was sucessfully tested in a rotating model test rig at peripheral speeds up to 200 m/s (5000 RPM). The temperature profile of a stationnary hot air jet traversed by the probe could be restituted with reasonable accuracy at jet passing frequencies up to 6 kHz.

Acknowledgements

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References

Collis, D.C., Williams, M.J., 1959, "Two dimensionnal convection from heated wires at low Reynolds number", Journal of Fluid Mechanics, Vol. 6, pages 357,384.

Hojstrup, J., Rasmussen, K., Larsen, S.E., 1976 "Dynamic calibration of temperature wires in still air"

Ji Ryong Cho, Kyung Chun Kim, 1993 "A simple high performance cold-wire thermometer", Meas. Sci. Technol., Vol.4, pages 1346–1349.

LaRue,J.C., Deaton, T., Gibson, C.H., 1974, "Measurement of high-frequency turbulent temperature", Review of scientific instruments, Vol. 46 no 6, p.757,764.

Oppenheim, A.V., Willsky, A.S, Young, I.T., 1983 "Signals and systems", Prentice/Hall International editions.

Parantheon, P., Petit, C., Lecordier, J.C., 1982 "The effect of thermal prong-wire interaction on the response of a cold wire in gaseous flow", Journal of Fluid Mechanics, Vol. 124 pages 457–473.

Rabiner, L.R., Gold, B., 1975, "Theory and application of digital processing", Prentice-Hall.

Sieverding, C.H., Vanhaeverbeek, C., Schulze, G., 1992 "An opto-electronic data transmission system for measurements on rotating turbomachinery components", ASME paper 92-GT-337.

Tsuji, T., Nagano, Y., Tagawa, M., 1992 "Frequency response and instataneous temperature profile of cold-wire sensors for fluid temperature fluctuation measurements", Experiments in Fluids.

Weeks, A.R., Beck, J.K., Joshi, M.L., 1988, "Response and compensation of temperature sensors", Journal of Physics and Scientific Instrumentation.

Figures

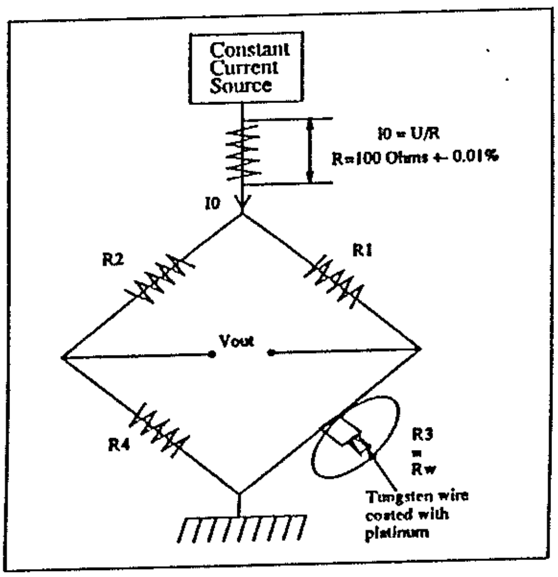

Figure 1: Principle of resistance thermometer

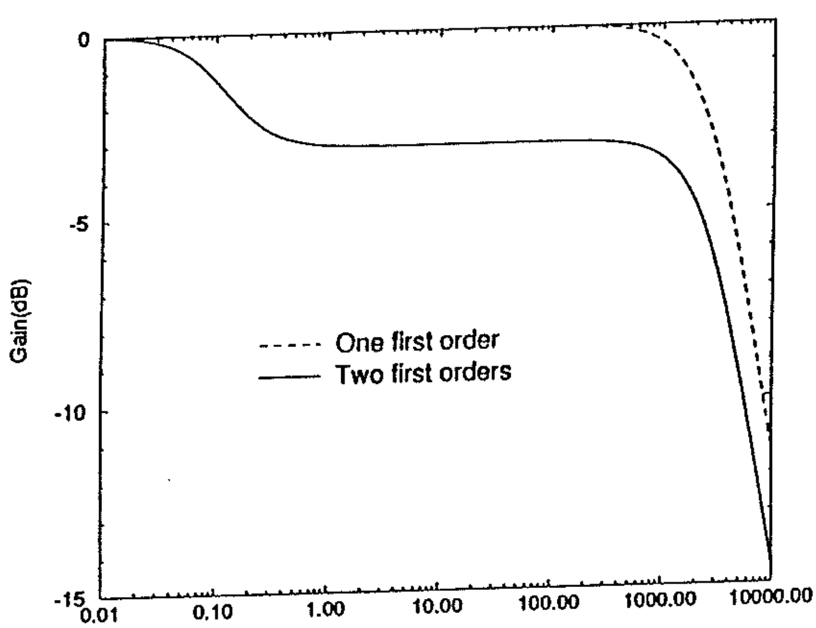

Figure 2: Transfer function using two first orders

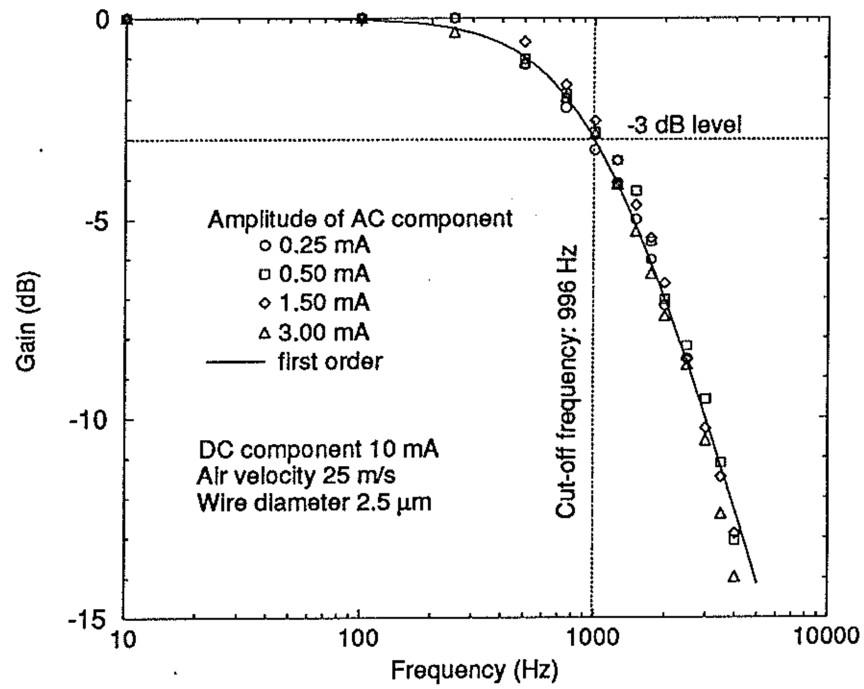

Figure 3: Gain obtained by electrical heating for several current amplitudes

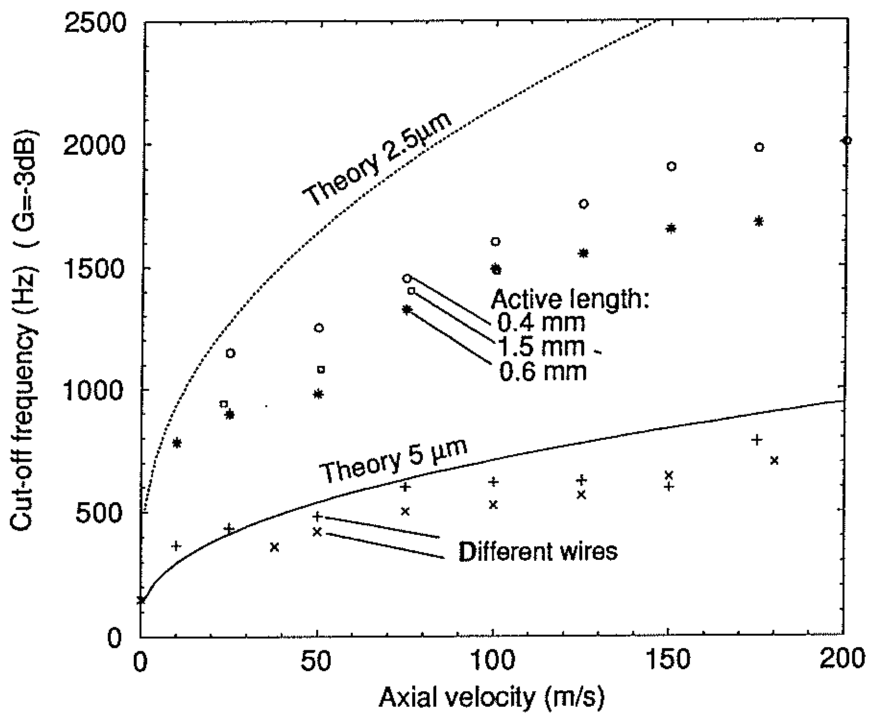

Figure 4: Cut-off frequency in function of air velocity

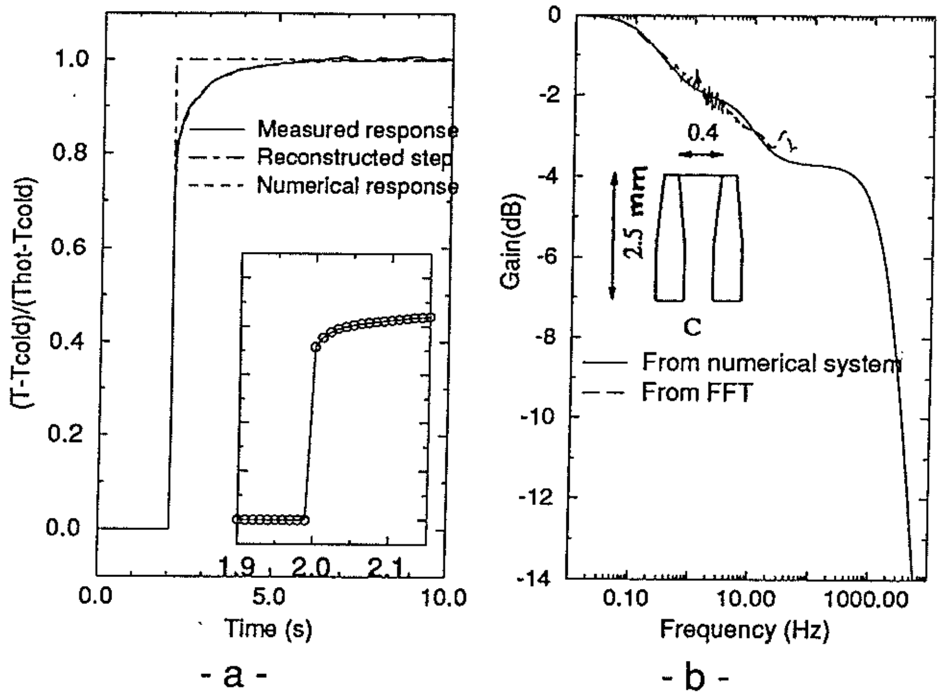

Figure 5: Step test and prong transfer function

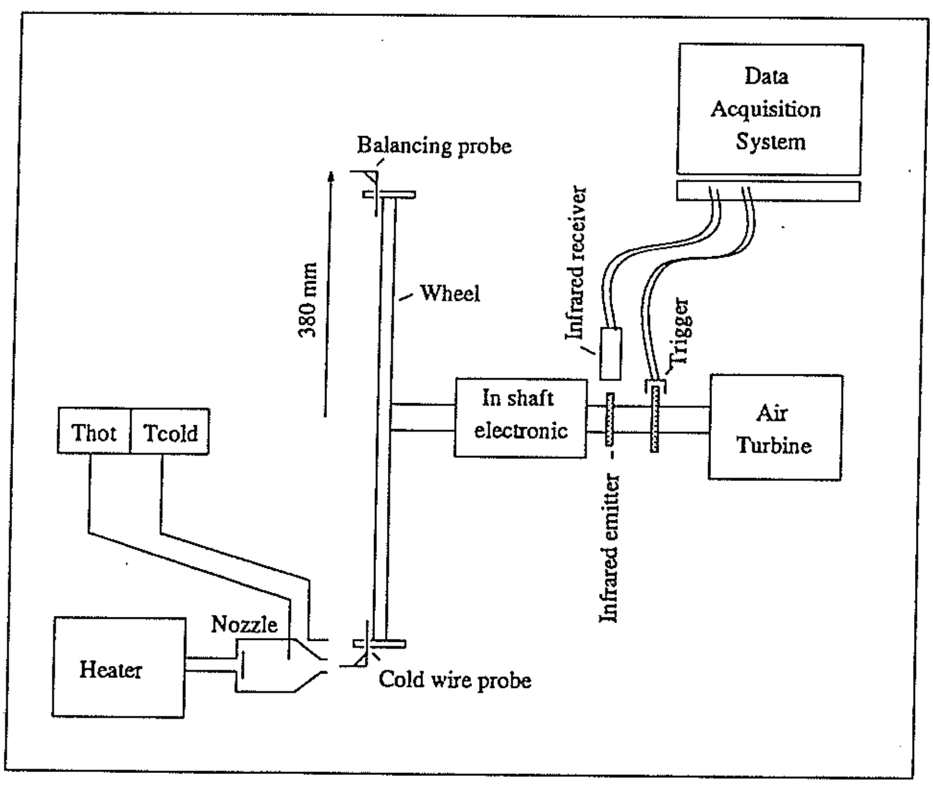

Figure 6: Set-up for measurements in rotation

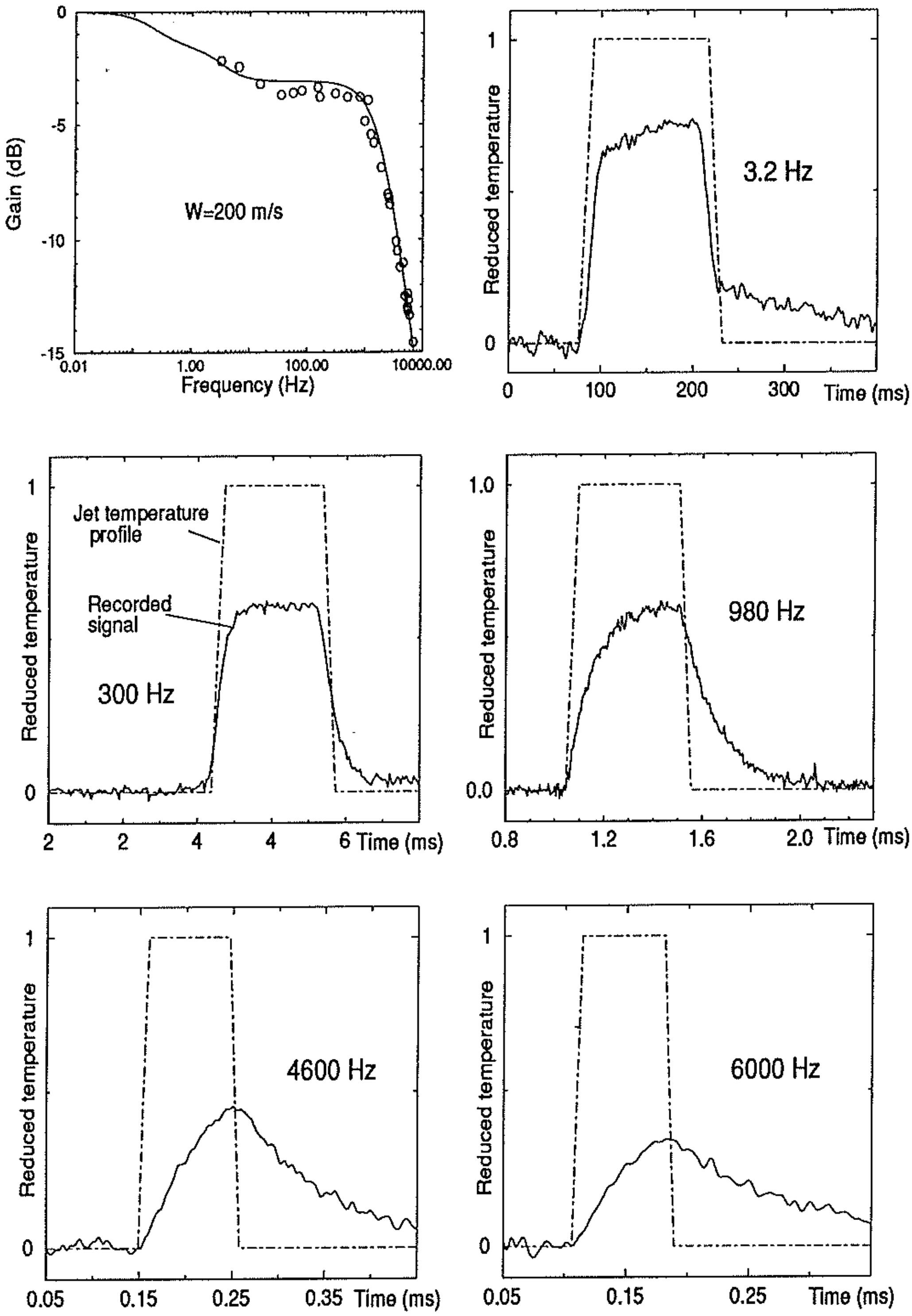

Figure 7: Temperature signals measured in rotation

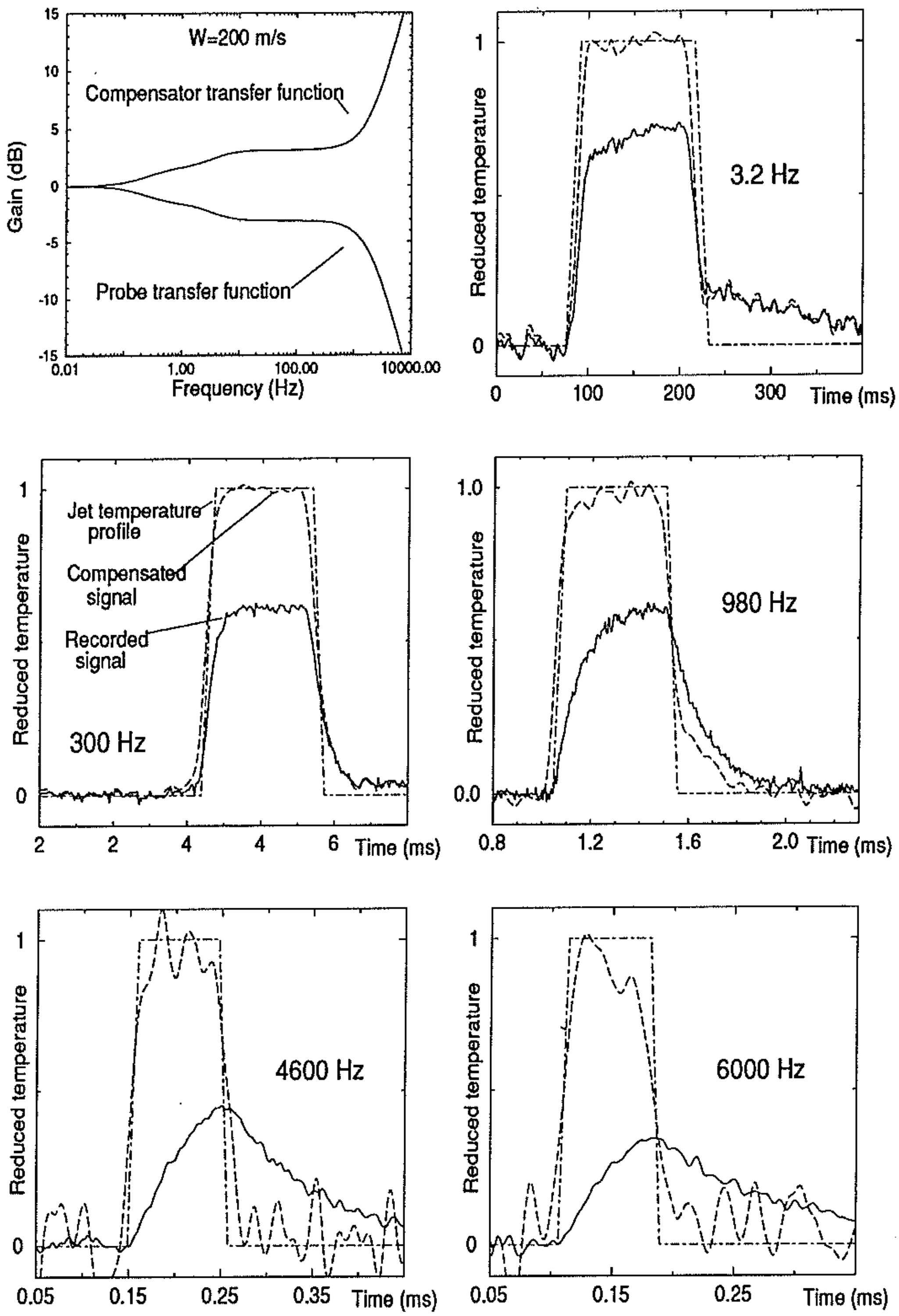

Figure 8: Compensated signals